Robust Fixed Interval Satellite Range Scheduling

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Abstract— **The Satellite Range Scheduling problem has been solved in previous work by the authors. However, real scenarios may involve contingencies on the satellites, the ground stations or the communication link, which in practice can be translated as communication requests eventually being dropped from the schedule with a certain probability. Compared to existing sub-optimal approaches which add back-up passes to a nominal schedule, robust scheduling finds the schedule with maximal expected performance. Robust schedules are not necessarily free of conflicts, conversely to optimal schedules, and thus finding the robust schedule poses increased complexity. The authors investigate the tractability bounds for the case where these requests have fixed start and end times, different priorities, and different failure probabilities, and provide a linear time algorithm for obtaining the robust schedule in scenarios with a single scheduling entity, laying the foundations for studying more complex cases.**

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1. INTRODUCTION

Satellite missions aim to establish communications between their mission control center and its associated satellite, but this can only be done through a network of ground stations. Whereas mission control centers have continuous access to the ground stations via terrestrial networks, the times when potential communication can occur between the satellites and the ground stations are defined by passes. These passes are defined by the visibility time windows (line of sight) of a set of satellites (traveling through their orbits) over a set of ground stations (which move with the surface of the Earth). The missions, depending on their requirements, could need the whole time windows or just a portion of them. The problem of maximizing some metric (e.g. total duration of passes in the schedule) on the allocation of these passes is known in existing literature as the *satellite range scheduling (SRS) problem*. For a more detailed description see ref. [1].

The SRS problem has been solved optimally by the authors [1], [2], as opposed to existing alternatives which are sub-optimal (see for example [3], [4], [5], [6]). But regardless of the optimality of these solutions, passes on the schedule could be dropped if whether the associated satellite or ground station fail to establish communication due to factors not included or that changed from what was included in the scheduling problem (weather, maintenance, etc.).

Existing literature has provided sub-optimal solutions for robust SRS for a single scheduling entity through different approaches, for example based on fuzzy logic [7] or adding backup passes to an initial schedule [8]. In this paper we focus on the case where communication intervals have fixed start and end times. We model the uncertainty by introducing failure probabilities associated to the requests, so that the schedule which is actually executed is different from the calculated one. We prove that this problem generalizes the SRS problem, and we provide the main contribution of the paper: an algorithm for calculating the schedule with maximal expected metric through incrementally building a schedule for a single scheduling entity. Since more general cases of SRS have been proven to be transformable into instances of the fixed interval case [1], we consider this work as the first step for tackling more complex cases of robust SRS.

The paper is organized as follows. In Section 2 we review the SRS problem. In Section 3 we present the robust SRS problem, and provide its complexity bounds. In Section 4 we provide a linear time algorithm for a single scheduling entity. In Section 5 we show a detailed example on the application of this algorithm and some simulations to illustrate its results compared to traditional algorithms. In Section 6 we provide the conclusions of the research.

2. SATELLITE RANGE SCHEDULING

Let $S = \{s_i\}$ be a set of satellites, and $G = \{g_h\}$ a set of ground stations. The motion dynamics of these two kinds of entities will generate visibility time windows, or passes, when lines of sight among the different entities exist [9]. We consider a scheduling period T starting at t_0, so that $t \in [t_0, t_0 + T]$.

Let a *pass* p_l be a tuple modeling a visibility time window from a start time t_s to an end time t_e between the satellite s_i and the ground station g_h, with an assigned priority w_l:

$$p_l = (s_i, g_h, t_{\mathrm{s}_l}, t_{\mathrm{e}_l}, w_l). \quad (1)$$

The term $w_l \in \mathbb{R} \cap [0, 1]$ characterizes the weight or *priority* associated to that request, normalized between 0 and 1. Let $P = \{p_l\}$ be a set of $|P| = N$ passes.

Remark 1: We assume that a ground station can only communicate to one satellite at the same time and vice-versa.

These resource constraints are defined via *conflicts among*

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passes. We consider that a pass p_l is conflicting with an earlier start time pass p_m if they are time overlapping and both are associated with either the same satellite or ground station:

$$\begin{aligned}\phi(p_l, p_m) = 1 \Leftrightarrow \\ &\{(g(p_l) = g(p_m)) \vee (s(p_l) = s(p_m))\} \wedge \\ &\{t_{s_l} \in [t_{s_m}, t_{e_m}]\} \wedge \{l \neq m\},\end{aligned} \quad (2)$$

otherwise $\phi(p_l, p_m) = 0$.

We say that $P_{\text{sub}} \subset P$ is a *feasible schedule* if all its passes are non-conflicting:

$$P_{\text{sub}} \in \{P^{\text{f}}\} \;\Leftrightarrow\; \sum \phi(p_l, p_m) = 0 \;\; \forall p_l, p_m \in P_{\text{sub}}, \quad (3)$$

where $\{P^{\text{f}}\}$ is the set of all the feasible schedules.

Given a feasible schedule P_{f}, let the *metric* $\|\cdot\|_{\Sigma w}$ be:

$$\|P^{\text{f}}\|_{\Sigma w} = \sum_l w_l \;:\; w_l = w(p_l)\; \forall p_l \in P^{\text{f}}. \quad (4)$$

The objective of the *SRS problem* is finding a feasible schedule with maximal metric [1]:

$$P^* \triangleq \arg\max(\|P_{\text{sub}}\|_{\Sigma w})\; \forall P_{\text{sub}} \in \{P^{\text{f}}\}, \quad (5)$$

where $\{P^{\text{f}}\}$ is the set of feasible schedules.

In the remainder of the paper we will refer to this problem as the *Basic SRS Problem*. References [1], [2] present the optimal solution of this problem under the assumption that all the entities are 100% reliable, that is, there is no chance that a pass in the optimal schedule will fail in the future.

In the following section we describe the changes that we introduce in the formulation of the problem for taking into account uncertainty.

3. Robust Satellite Range Scheduling

We consider that a pass $p_l \in P$ can be dropped from the schedule with probability α_l. Let $P' \subset P$ be a sub-schedule of P, and let $\widetilde{P'} \subset P'$ be the *executed schedule* of P'. That is, $\widetilde{P'}$ is the set of passes which were not dropped from the schedule P' at the end of the scheduling window $t_0 + T$, either because of the failure probabilities or earlier start time conflicting passes being in this set. Then:

- P is the initial set of passes (problem definition).
- $P' \subset P$ is the selected schedule (proposed solution).
- $\widetilde{P'} \subset P'$ is the executed schedule (actual result).

Remark 2: We assume that these probabilities α_l are independent, and that they model all the factors in the system which introduce uncertainty: satellite or ground station related (α_{S},α_{G}), weather related (α_{W}), etc. Note that even if we introduce time-dependency (α_t) in the model of these probabilities, the start times of the passes are fixed, and thus the values of α_l would be fixed for p_l. Knowledge of the model generating these priorities e.g. $\alpha_l = f(\alpha_{\text{S}}, \alpha_{\text{G}}, \alpha_{\text{W}}, \alpha_{\text{t}})$ would simplify the problem. Therefore, assuming independence among these probabilities, given a known P', we have that the probability of a pass p_l to be in the executed schedule $\widetilde{P'}$ given that it is in the schedule P' is:

$$\begin{aligned}&\mathbb{P}\big(p_l \in \widetilde{P'} \mid p_l \in P'\big) = \\ &\qquad (1 - \alpha_l) \cdot \mathbb{P}\big(\nexists p_m \in \widetilde{P'} : \phi(p_l, p_m) = 1\big).\end{aligned} \quad (6)$$

From Remark 1 we have that the set $\widetilde{P'}$ has to be feasible $\widetilde{P'} \in \{P^{\text{f}}\}$.

Definition 1: We define the *expected metric of a schedule* P' as the expected value of the metric of the executed schedule:

$$\|P'\|_{\mathbb{E}} = \mathbb{E}\{\|\widetilde{P'}\|_{\Sigma w}\} = \sum_{p_l \in P'} w_l \cdot \mathbb{P}\big(p_l \in \widetilde{P'} \mid p_l \in P'\big). \quad (7)$$

Definition 2: The problem of *Robust Satellite Range Scheduling* can be stated as finding the *robust schedule* P^{R}, which is a schedule with maximal expected metric.

$$P^{\text{R}} \subseteq P,\;\; \nexists P_{\text{sub}} \subseteq P : \|P_{\text{sub}}\|_{\mathbb{E}} > \|P^{\text{R}}\|_{\mathbb{E}}. \quad (8)$$

Note that in contrast to the basic SRS problem we now should not restrict the space of possible solutions $P' \subset P$ to feasible schedules, since those dropped passes could allow later conflicting passes (if we include them in the schedule) to contribute to the metric.

Complexity of the Robust SRS Problem

In this section we study the tractability of finding the robust schedule.

Lemma 1: The Robust SRS problem generalizes the Basic SRS problem.

Proof: If $\alpha_l = 0\; \forall p_l \in P$, then from (6) we have that $\mathbb{P}\big(p_l \in \widetilde{P'} \mid p_l \in P'\big) = \mathbb{P}\big(\nexists p_m \in \widetilde{P'} : \phi(p_l, p_m) = 1\big)$, and a pass from P' will be in $\widetilde{P'}$ if there are no prior conflicting passes in $\widetilde{P'}$. Therefore, given any feasible schedule $P' \subset P : P' \in \{P^{\text{f}}\}$, we have $p_l \in P' \Rightarrow p_l \in \widetilde{P'}$, so that we can reduce the space of possible solutions to the space of feasible schedules (since no pass will be dropped), and thus $\|P^{\text{R}}\|_{\mathbb{E}} = \|P^*\|_{\Sigma w}$ if $\alpha_l = 0\; \forall p_l \in P$. □

Theorem 1: The robust fixed interval satellite range scheduling problem is $\mathcal{NP}$-hard.

Proof: In ref. [1] we show that the decision version of the basic SRS problem is $\mathcal{NP}$-complete, and thus the optimization version is $\mathcal{NP}$-hard. From Lemma 1, robust SRS has to be at least as complicated as basic SRS, and therefore Robust SRS is $\mathcal{NP}$-hard. □

The problem is simplified however if we consider a single scheduling entity. In the rest of the paper we focus on this case.

4. Restricted Robust SRS Problem

In this section we provide an algorithm for finding the robust schedule for instances of the problem with a single scheduling entity.

We assume that the set P is ordered by increasing start time, i.e. $P = \{p_1, p_2, \ldots, p_N\}$ such that $t_s(p_i) < t_s(p_j) \Leftrightarrow i < j$.

We denote the subset $P_l \subset P$ associated to p_l as the set of passes $\{p_l, p_{l+1}, \ldots, p_N\} \subset P$, such that $p_l \in P_l$ and $p_k \in P_l \Leftrightarrow t_s(p_l) < t_s(p_k)$. We will use this notation in the remainder of the paper.

Definition 3: We define the *set of later non-conflicting passes* for the pass p_l as the subset D_l whose passes have a later start time than p_l and which are not conflicting with it. Then:

$$\begin{aligned} \forall p_l \in P,\ \exists D_l \subset P_l\ :\ \forall p_d \in P_l,\ \ p_d \in D_l \Leftrightarrow \\ t_s(p_l) < t_s(p_d) \wedge \phi(p_d, p_l) = 0, \end{aligned} \tag{9}$$

so that if $\nexists p_d \in P_l : t_s(p_l) < t_s(p_d) \wedge \phi(p_d, p_l) = 0$ then $D_l = \emptyset$.

Proposition 1: Let P be a set of passes sorted by increasing start time. If there is a single scheduling entity ($|G| = 1$ or $|S| = 1$), we have that $D_l \cap P_l = P_x\ \forall p_l \in P$, where $p_x \in P_l$ is the pass with earliest start time not conflicting with p_l.

Proof: If whether $|G| = 1$ or $|S| = 1$, then from (2):

$$\phi(p_m, p_l) = 1 \Leftrightarrow t_{s_m} \in [t_{s_l}, t_{e_l}]\ \forall p_l, p_m \in P,\ l \neq m, \tag{10}$$

so that two passes being time-overlapping is a sufficient condition for them to be conflicting.

Therefore, given a pass $p_l \in P$, if there is a pass p_m with later start time and not conflicting with it, then any pass with later start time than p_m is not conflicting with p_l:

$$\begin{aligned} \forall p_l, p_m, p_k \in P\ :\ t_s(p_l) < t_s(p_m) < t_s(p_k), \\ \phi(p_m, p_l) = 0\ \Rightarrow \phi(p_k, p_l) = 0. \end{aligned} \tag{11}$$

Let $p_x \in P_l$ be the pass with the earliest start time in D_l, then $\forall p_m \in P_l$ we have that $t_s(p_x) < t_s(p_m) \Rightarrow \phi(p_m, p_l) = 0$, thus $p_m \in D_l$, and therefore $D_l \cap P_l = P_x$. □

Note that this result does not necessarily hold if both $|G|$ and $|S|$ are greater than one.

Lemma 2: The *expected metric of a schedule* $P' = \{p_y, \ldots, p_z\} \subset P$ with a single scheduling entity can be computed recursively in $\mathcal{O}(\mathcal{N})$ as $\|P'\|_{\mathbb{E}} = \|P'_y\|_{\mathbb{E}}$ where:

$$\|P'_l\|_{\mathbb{E}} = \begin{cases} (1-\alpha_l)(w_l + \|D_l \cap P'_{l+1}\|_{\mathbb{E}}) + \\ \quad \alpha_l \|P'_{l+1}\|_{\mathbb{E}}, & p_l \in P', \\ \|P'_{l+1}\|_{\mathbb{E}}, & p_l \notin P', \end{cases} \tag{12}$$

for all $p_l \in P'$, and with $\|P'_l\|_{\mathbb{E}} = 0\ \forall l > z$ for consistency of the algorithm.

Proof: Let $p_x \in P'$ be the pass with earliest start time which is not conflicting with p_l, such that $t_s(p_l) < t_s(p_x)$, and let $P'_x = P_x \cap P'$ and $P'_l = P_l \cap P'$. Since there is a single scheduling entity, we have from Prop. 1 that $D_l \cap P'_l = P'_x$.

We calculate the expected metric of the sub-schedule P'_l. For simplicity of notation we will work with P, P_l and P_x (instead of P', P'_l, and P'_x), since this will allow us to keep all the indexes for the passes $k = 1, 2, \ldots, N$. There are two possible cases: $p_l \in P_l$ and $p_l \notin P_l$.

(i) If $p_l \in P_l$, from (7) we have:

$$\|P_l\|_{\mathbb{E}} = \sum_{k=l}^{N} w_k \cdot \mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \in P_l\big). \tag{13}$$

Since $p_l \in P_l$, we have that either $p_l \in \widetilde{P}_l$ or $p_l \notin \widetilde{P}_l$, with probabilities $\mathbb{P}\big(p_l \in \widetilde{P}_l \mid p_l \in P_l\big) = (1-\alpha_l)$ and $\mathbb{P}\big(p_l \notin \widetilde{P}_l \mid p_l \in P_l\big) = \alpha_l$. Therefore:

$$\begin{aligned} &\mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \in P_l\big) = \\ &\quad (1-\alpha_l) \cdot \mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \in \widetilde{P}_l\big) + \\ &\quad \alpha_l \cdot \mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \notin \widetilde{P}_l\big). \end{aligned} \tag{14}$$

For the calculation of $\mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \in \widetilde{P}_l\big)$ we consider three sets of passes: (1) the pass p_l ($k = l$), (2) the set of later passes conflicting with it ($l + 1 \leqslant k \leqslant x - 1$), and (3) the rest ($x \leqslant k \leqslant N$). Since we are assuming that p_l is in the executed schedule, for the pass p_l (1) it is easy to see that this probability is one; for the same reason the passes which are conflicting with p_l (2) cannot be in the executed schedule, so that the probability is zero; and for the rest of passes (later non-conflicting) (3), none of the passes prior to the first of this set will be in the final schedule except p_l, which is not conflicting with any pass in this set, and thus we simply consider this set for the calculation.

For calculating $\mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \notin \widetilde{P}_l\big)$ we only consider two sets: the pass p_l, and given that p_l is not in the executed schedule, the rest of passes. Also for this reason it is easy to see that this probability is zero for p_l; and that again for the rest of passes we simply consider this set for the calculation of the priority.

Therefore the probabilities in (14) are:

$$\begin{aligned} &\mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \in \widetilde{P}_l\big) = \\ &\quad \begin{cases} 1, & k = l, \\ 0, & l < k < x, \\ \mathbb{P}\big(p_k \in \widetilde{P_x} \mid p_k \in P_x\big), & x \leqslant k \leqslant N, \end{cases} \end{aligned} \tag{15}$$

$$\begin{aligned} &\mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \notin \widetilde{P}_l\big) = \\ &\quad \begin{cases} 0, & k = l, \\ \mathbb{P}\big(p_k \in \widetilde{P_{l+1}} \mid p_k \in P_{l+1}\big), & l < k \leqslant N. \end{cases} \end{aligned} \tag{16}$$

For consistency, if $D_l = \emptyset$ then we would only consider the cases $k = l$ and $l < k \leqslant N$ (or simply that $x = N + 1$) in (15). Developing (13) throughout (14-16) we have:

$$\begin{aligned} \|P_l\|_{\mathbb{E}} = (1-\alpha_l) w_l + \\ (1-\alpha_l) \sum_{k=x}^{N} w_k \cdot \mathbb{P}\big(p_k \in \widetilde{P_x} \mid p_k \in P_x\big) + \\ \alpha_l \sum_{k=l+1}^{N} w_k \cdot \mathbb{P}\big(p_k \in \widetilde{P_{l+1}} \mid p_k \in P_{l+1}\big). \end{aligned} \tag{17}$$

And regrouping elements in (17) we obtain (12) for the case $p_l \in P_l$.

(ii) If $p_l \notin P_l$, from (7) we have:

$$\|P_l\|_{\mathbb{E}} = \sum_{k=l}^{N} w_k \cdot \mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \notin P_l\big). \tag{18}$$

In this case, given that $p_l \notin P_l$, we have that $p_l \notin \widetilde{P}_l$. Therefore:

$$\|P_l\|_{\mathbb{E}} = \sum_{k=l}^{N} w_k \cdot \mathbb{P}\big(p_k \in \widetilde{P}_l \mid p_k \in P_l,\ p_l \notin \widetilde{P}_l\big). \qquad (19)$$

Substituting (16) in (19) we obtain (12) for the case $p_l \notin P_l$, completing the proof.

The computation of $\|P\|_{\mathbb{E}}$ requires the iterative calculation of the expected metric of the sub-schedules $P_N, P_{N-1}, \ldots, P_1$, which therefore takes $\mathcal{O}(\mathcal{N})$. □

In summary, Lemma 2 states that we can calculate the metric of schedules with a single scheduling entity in linear time in the number of passes, examining the passes by decreasing start time. For each pass examined, there are a set of later passes (later start times), and a set of later non-conflicting passes. Since the calculation is iterative, the metric of the schedules associated to later passes will have been already calculated. If the pass examined failed during execution, the final metric would be that of the set of latter passes; and if successful, the final metric would be that of the set of later non-conflicting passes plus the priority of the examined pass. Finally we calculate the expected metric by combining these two metrics with the probability of failure for this pass.

This result can be used for the calculation of the robust schedule, which is the main contribution of the paper. In this case we also examine the set of passes by decreasing start time, but we will only add the examined pass to the final schedule if the calculated metric is greater than that associated to the next pass (just previously calculated). This is proven in Theorem 2.

Theorem 2: The *robust schedule* for a set of passes P with a single scheduling entity can be computed recursively in $\mathcal{O}(\mathcal{N})$ as $P^{\mathsf{R}} = P_1^{\mathsf{R}}$ where:

$$\|P_l^{\mathsf{R}}\|_{\mathbb{E}} = \max \begin{cases} (1-\alpha_l)(w_l + \|D_l \cap P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}) + \\ \quad \alpha_l \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}, \\ \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}, \end{cases} \qquad (20)$$

$$P_l^{\mathsf{R}} = \begin{cases} \{p_l\} \cup P_{l+1}^{\mathsf{R}}, & \|P_l^{\mathsf{R}}\|_{\mathbb{E}} \geqslant \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}, \\ P_{l+1}^{\mathsf{R}}, & \text{otherwise}, \end{cases} \qquad (21)$$

for all $p_l \in P$, and where P_l^{R} is the robust schedule for P_l.

Proof: We prove optimality by induction. Let $P^{\mathsf{R}} \subset P$ and $P_l^{\mathsf{R}} = P_l \cap P^{\mathsf{R}}$, and since $p_l \notin D_l$ we have that $D_l \cap P_l^{\mathsf{R}} = D_l \cap P_{l+1}^{\mathsf{R}}$. For a single scheduling entity we have from Prop. 1 that $D_l \cap P_{l+1}^{\mathsf{R}} = P_x^{\mathsf{R}}$, where p_x is the first pass non-conflicting with p_l which is in P_{l+1}^{R}. From Lemma 2, the expected metric of the schedule $\|P_l^{\mathsf{R}}\|_{\mathbb{E}}$ is maximal if $\|P_x^{\mathsf{R}}\|_{\mathbb{E}}$ and $\|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}$ are maximal, which is equivalent to stating that P_l^{R} is robust for P_l if P_x^{R} and P_{l+1}^{R} are robust for P_x and P_{l+1} respectively. Since p_N has no latter conflicting passes, it is easy to see that $P_N^{\mathsf{R}} = \{p_N\}$ is robust for P_N. Therefore $P_1^{\mathsf{R}} = P^{\mathsf{R}}$ is robust.

The calculation of the robust schedule is iterative ($l = N, N-1, \ldots, 1$), and thus the calculation of P^{R} takes $\mathcal{O}(\mathcal{N})$. The pass p_l is included in P_l^{R} if that increases the expected metric of the schedule, that is, if $(1-\alpha_l)(w_l + \|D_l \cap P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}) + \alpha_l \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}} \geqslant \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}$. □

Therefore, *the Robust SRS problem with a single ground station or satellite and a set of N passes $P = \{p_1, p_2, ..., p_N\}$ with associated weights w_i, failure probabilities $\alpha_i \in [0, 1]$ and fixed times $(t_{\mathrm{s}_i}, t_{\mathrm{e}_i})\ \forall p_i \in P$ can be solved in $\mathcal{O}(\mathcal{N})$.*

If the failure probabilities were unknown, we could consider the worst case, which is $\alpha_l \to 1\ \forall p_l \in P$. Under this assumption, if we calculate the robust schedule (Thm. 2), we have that the first line of (20) coincides with the second one: $\alpha_l = 1\ \forall p_l \in P \Rightarrow (1-\alpha_l)(w_l + \|D_l \cap P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}) + \alpha_l \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}} = \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}$, and thus $\|P_l^{\mathsf{R}}\|_{\mathbb{E}} = \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}}$, so that we always add the pass under consideration to the schedule (21), that is: $\|P_l^{\mathsf{R}}\|_{\mathbb{E}} = \|P_{l+1}^{\mathsf{R}}\|_{\mathbb{E}} \Rightarrow P_l^{\mathsf{R}} = \{p_l\} \cup P_{l+1}^{\mathsf{R}}\ \forall p_l \in P$. Therefore, $P^{\mathsf{R}} = P$ if $\alpha_l \to 1\ \forall p_l \in P$.

5. Application Examples

In this section we show the importance of taking uncertainty into account through an example detailing the creation of the robust schedule, and a simulation for showing the performance of the algorithm in practical scenarios. We include the results of another two algorithms generally used in SRS: *optimal* algorithm for basic SRS [2] (for which the selected schedule is P^*), and the *greedy earliest-start-time* algorithm (see for example [10], for which the selected schedule is P).

Schedule Computation

We show a simple example for the calculation of the robust schedule for a single ground station. The set of passes $P = \{p_1, p_2, \ldots, p_6\}$ with associated priorities $w_1 = 0.2$, $w_2 = 0.9$, $w_3 = 0.5$, $w_4 = 0.4$, $w_5 = 0.1$ and $w_6 = 0.7$ is represented in Figure 1, where we consider $\alpha_l = 0.5\ \forall p_l \in P$.

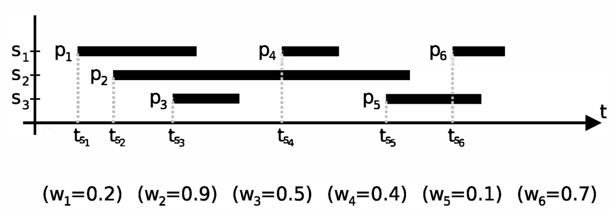

Figure 1. Set of passes for a single ground station.

Note that since all the passes are associated to the same scheduling entity (ground station), the scheduling resource (satellite) associated to each pass is irrelevant as time overlap is a sufficient and necessary condition for two passes to be conflicting (2).

We follow the algorithm from Thm. 2 for the calculation of the robust schedule. We simplify the notation so that $\|P_l^{\mathsf{R}}\|_{\mathbb{E}} = \mu_l$:

Pass p_6: $P_6^{\mathsf{R}} = \{p_6\}$ and $\mu_6 = (1-\alpha_6)w_6 = 0.35$.

Pass p_5: $P_5^{\mathsf{R}} = \{p_6\}$ and $\mu_5 = 0.35$. If we had included p_5 in

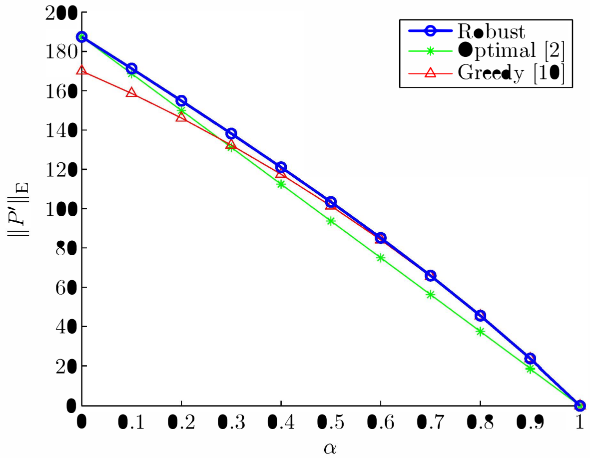

Figure 2. Theoretical expected metrics for the scenario 1.

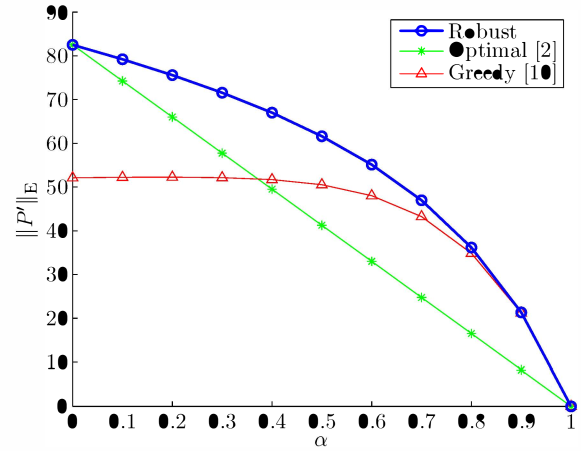

Figure 4. Theoretical expected metrics for the scenario 2.

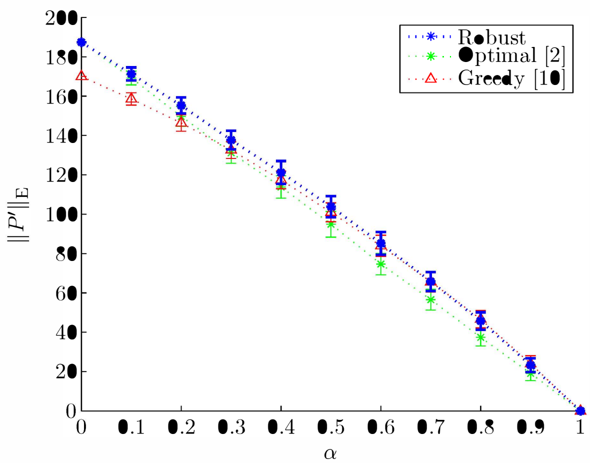

Figure 3. Simulated expected metrics for the scenario 1.

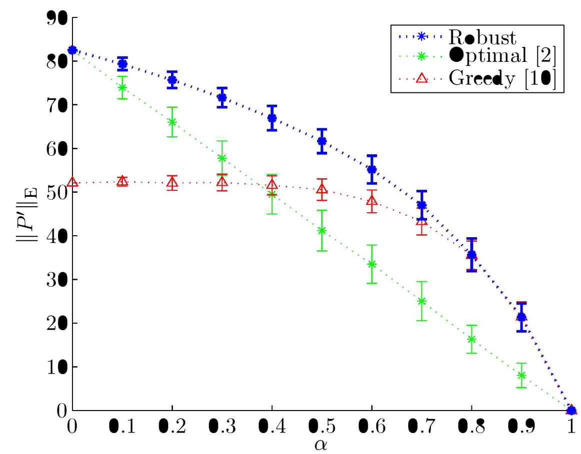

Figure 5. Simulated expected metrics for the scenario 2.

the schedule then the expected metric would be $(1-\alpha_5)w_5 + \alpha_5\mu_6 = 0.18$, which is smaller than μ_6.

Pass p_4: $P_4^{\mathrm{R}} = \{p_4, p_6\}$, and $\mu_4 = (1-\alpha_4)w_4 + \mu_5 = 0.55$. Since p_4 has not any pass conflicting with it in P_5^{R} we add it to the schedule.

Pass p_3: $P_3^{\mathrm{R}} = \{p_3, p_4, p_6\}$ and $\mu_3 = (1-\alpha_3)w_3 + \mu_4 = 0.8$ following the same reasoning as for P_4^{R}.

Pass p_2: $P_2^{\mathrm{R}} = \{p_2, p_3, p_4, p_6\}$ and $\mu_2 = 1.025$. The first pass in $D_2 \cap P_3^{\mathrm{R}}$ is p_6. We add the pass to the schedule, so the expected metric is $(1-\alpha_2)(w_2 + \mu_6) + \alpha_2\mu_3 = 1.025$, which is bigger than μ_3.

Pass p_1: $P_1^{\mathrm{R}} = \{p_2, p_3, p_4, p_6\}$ and $\mu_1 = 1.025$. The first pass in $D_1 \cap P_2^{\mathrm{R}}$ is p_4. If we had added p_1 to the schedule the expected metric would be $(1-\alpha_1)(w_1 + \mu_4) + \alpha_1\mu_2 = 0.8875$, which is smaller than μ_2.

Thus $P^{\mathrm{R}} = \{p_2, p_3, p_4, p_6\}$ and $\|P^{\mathrm{R}}\|_{\mathbb{E}} = 1.025$. The optimal algorithm yields $P^* = \{p_2, p_6\}$, with $\|P^*\|_{\Sigma w} = 1.6$ and $\|P^*\|_{\mathbb{E}} = 0.8$, and the greedy earliest-start-time algorithm yields the metric $\|P\|_{\mathbb{E}} = 0.67$.

Simulations

We implemented the three algorithms (optimal, greedy and robust) in MATLAB for evaluating their performance with different values for the failure probabilities. We consider the two scenarios we introduced in ref. [2] but considering a single ground station and a fixed scheduling window of 14 days:

Scenario 1. Uncorrelated resources—One ground station, and five satellites in different low Earth orbits (LEO). Passes are given a random priority $w(p_l) = v_l/10 : v_l \sim$ U[1,10], and we assign the same failure probability to all the passes ($\alpha_l = \alpha\ \forall l$) varying α between 0 and 1 in steps of 10^{-1}.

Scenario 2. Correlated resources—One ground station, and five satellites with identical orbits. This second scenario corresponds to the worst case for the presented algorithm, as this case yields the maximum number of conflicts. Passes are given a random priority $w(p_l) = v_l/10 : v_l \sim$ U[1,10], and we assign the same failure probability to all the passes ($\alpha_l = \alpha\ \forall l$) varying α between 0 and 1 in steps of 10^{-1}.

We calculate the expected metric of the three algorithms $\|P^*\|_{\mathbb{E}}$ (optimal), $\|P\|_{\mathbb{E}}$ (greedy) and $\|P^{\mathrm{R}}\|_{\mathbb{E}}$ (robust) calculated through the algorithm in Lemma 2. Results are shown in Figs. 2 and 4 for scenarios 1 and 2 respectively.

Alternatively, we simulate 100 realizations of $\|\widetilde{P^*}\|_{\Sigma w}$, $\|\widetilde{P}\|_{\Sigma w}$ and $\|\widetilde{P^{\mathrm{R}}}\|_{\Sigma w}$ through (6) and (4) for the same range of values of α, for which we show their confidence intervals in Figs. 3 and 5.

These results show that $\|P^*\|_{\mathbb{E}} \leqslant \|P^{\mathrm{R}}\|_{\mathbb{E}}$ and $\|P\|_{\mathbb{E}} \leqslant \|P^{\mathrm{R}}\|_{\mathbb{E}}$, with the biggest differences in metric in the scenario where positions of the ground stations and orbits of the satellites are respectively highly correlated.

6. CONCLUSIONS

We have provided the tractability bounds for the robust fixed interval satellite range scheduling problem, which considers that communication links among ground stations and satellites may fail with a certain probability. We have presented an algorithm for finding the robust schedule for instances of the problem with a single scheduling entity in linear time. This robust schedule maximizes the expected performance.

Immediate applications of this work are on the scheduling of unreliable satellites in a ground station, or in the scheduling of an imaging satellite taking into account potential weather/visibility conditions. Future work includes tackling more general cases through the models presented in this paper.

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