

We develop a mathematical framework for quantifying and understanding multidimensional frequency modulations in digital images. We begin with the widely accepted definition of the instantaneous frequency vector (IF) as the gradient of the phase and define the instantaneous frequency gradient tensor (IFGT) as the tensor of component derivatives of the IF vector. Frequency modulation bounds are derived and interpreted in terms of the eigendecomposition of the IFGT. Using the IFGT, we derive the ordinary differential equations (ODEs) that describe image flowlines. We study the diagonalization of the ODEs of multidimensional frequency modulation on the IFGT eigenvector coordinate system and suggest that separable transforms can be computed along these coordinates. We illustrate these new methods of image pattern analysis on textured and fingerprint images. We envision that this work will find value in applications involving the analysis of image textures that are nonstationary yet exhibit local regularity. Examples of such textures abound in nature